# Norbert Wiener

*David Jerison and Daniel Stroock*

Through the week of October 8–14, 1994, a conference entitled "The Legacy of Norbert Wiener: A Centennial Symposium" was held at the Massachusetts Institute of Technology. The idea, of course, was to honor Norbert Wiener on this occasion and to review his contributions to mathematics, science, and engineering, but a deeper purpose was to bring out the significance of the interactions among these disciplines in contemporary research, as illustrated in the work of Wiener himself and others influenced by that work. We quote from the program:

[This symposium] begins with talks on current research in the areas of his fundamental contributions to mathematics. It continues with speakers representing a variety of disciplines with strong and growing relationships to mathematics. Finally, throughout the week there are talks devoted to Dr. Wiener's intellectual development and his profound influence on his colleagues at MIT and elsewhere.

An important goal of this symposium is to alert the mathematical, scientific, and engineering community to new opportunities for interactions between mathematics and other disciplines.

The symposium was sponsored by the Massachusetts Institute of Technology and the American Mathematical Society, with financial support from Henry Singleton, the Massachusetts Institute of Technology, the Sloan Foundation, and the National Science Foundation.

The proceedings of this conference will be published by the American Mathematical Society. The following article is a selection of excerpts from the biography printed for the program of the symposium, which was prepared by David Jerison and Daniel Stroock.

## Norbert Wiener

Norbert Wiener received his undergraduate degree in mathematics from Tufts University in 1909 at age fourteen. He first attempted graduate school in zoology at Harvard. He then spent a dismal year at Cornell in philosophy before returning to Harvard for a third try. He wrote his Ph.D. dissertation on the theories of Schroeder, Whitehead, and Russell and was granted his Ph.D. degree in philosophy at age eighteen. Although he claims to have found the work easy, he also admits that later "under Bertrand Russell in England, I learned that I missed almost every issue of true philosophical significance."

After graduation Wiener sailed for England on a travel grant to study at Cambridge. There he discovered a different breed of student who accepted his eccentricities and thrived on intellectual discussion. During that year he met another expatriate, T. S. Eliot, and they exchanged books and philosophical ideas. Wiener credits Russell with persuading him to learn some more genuine mathematics and acquainting him with the work of Einstein. But he was most inspired by G. H. Hardy, whom he called his "master in mathematical training". Hardy introduced him

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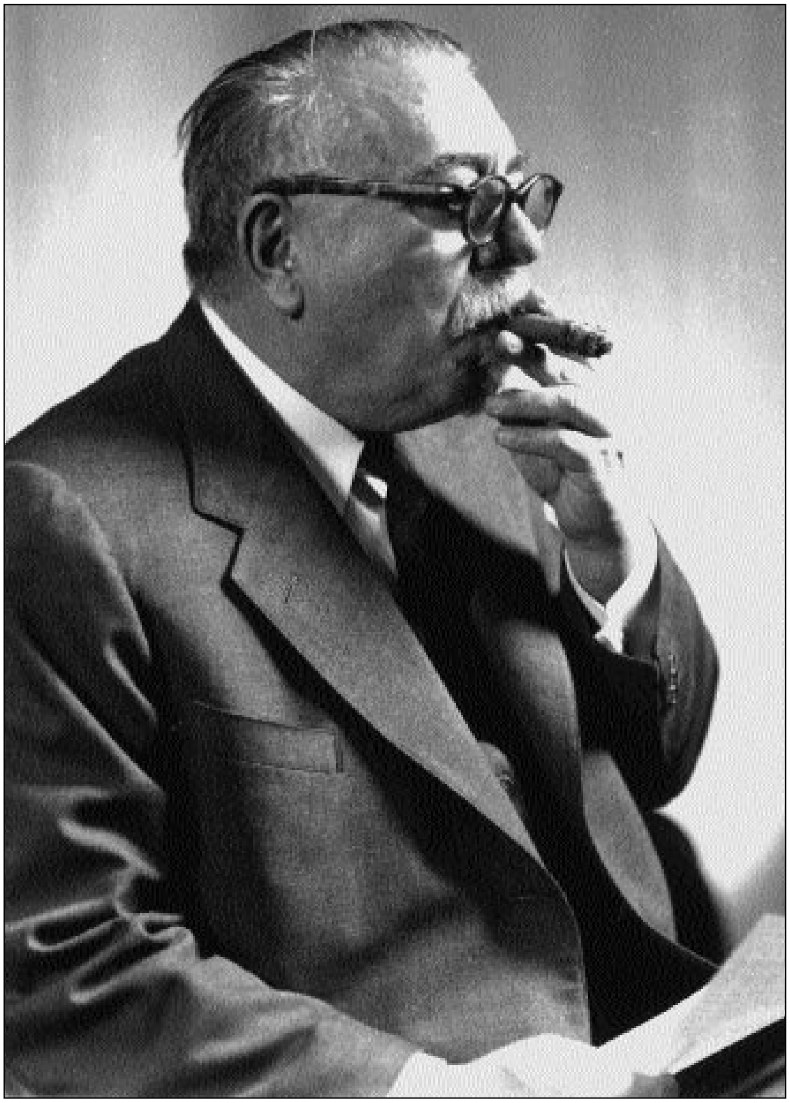
properly to complex variables and to the Lebesgue integral, topics that would play a major role in his later career.

Despite the importance of Hardy's influence, Wiener came to view Hardy's renowned condescension toward applications as "pure escapism". In their later encounters, Wiener bridled at Hardy's suggestion that Wiener's beautiful work on harmonic analysis was motivated solely by the internal aesthetics of mathematics and not by applications. In keeping with his deep and abiding interest in applications, Wiener believed that mathematicians cannot ignore the outside world and must both apply mathematics and bear the moral responsibility for applications. This conviction would become even more pronounced as time passed. Indeed, Wiener has had the last laugh: even Hardy's beloved number theory has applications to telecommunications, cryptography, and computer science.

Because Russell was planning to spend the spring semester at Harvard, Wiener decided to finish his postdoctoral year at Göttingen, home to such mathematical luminaries as David Hilbert and Edmund Landau as well as the philosopher Edmond Husserl. After Göttingen, he returned to England, hoping to spend the academic year 1914–1915 at Cambridge again. However, he found the university effectively shut down by the war and decided to return to America, where he had difficulty securing an academic position. Eventually, he received an invitation from Professor Oswald Veblen of Princeton to join Veblen's newly formed ballistics group at the Aberdeen Proving Ground in Maryland. This group's primary mission was to test new ordnance and to compute range tables which took into account the elevation angle, size of the charge, and other factors. Wiener seems to have enjoyed the direct practical application of mathematics in ballistics calculations, and his experience at Aberdeen served him well in his investigations of antiaircraft fire during World War II.

After the war, Wiener had hoped to follow Veblen back to Princeton, where Veblen was instrumental in assembling Princeton's soon-to-be-famous department of mathematics. The invitation never came. At about the same time, the fiancé of Wiener's sister Constance died in the influenza epidemic which swept the country after World War I. Constance's fiancé had been a budding mathematician, and after his untimely death Norbert received several mathematics books from his library. Thus, by accident, Wiener became acquainted with Volterra's *Theory of integral equations*, Osgood's *Theory of functions*, Lebesgue's book on the theory of integration, and Fréchet's treatise on the theory of functionals. Wiener claims that "For the first time I began to have a really good understanding of modern mathematics."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In 1919, after many odd jobs as far afield as journalism and engineering, Wiener's nomadic existence ended. Professor Osgood at Harvard obtained for Norbert an instructorship at MIT. Whether MIT's decision to hire Wiener was guided by phenomenal insider information or was just a fortuitous product of the "old boy network," there can be no doubt that Wiener's appointment was a gamble which paid off for both parties! Wiener remained at MIT until his retirement in 1960, and during that period he not only put MIT on the map mathematically, but he also played a profound part in the creation of the culture to which MIT owes much of its present fame and prestige.

## Mathematical Work at MIT

During his first dozen years at MIT, Wiener made his most astounding contributions to pure mathematics: He constructed Brownian motion, laid a new foundation for potential theory, and invented his generalized harmonic analysis.

The history of *Brownian motion* has taken some interesting twists and turns. The name honors the nineteenth century botanist Robert Brown, who reported that pollen and many types of inorganic particles suspended in water perform a strange St. Vitus dance. Brown refuted some facile explanations of this motion, although debate still raged over whether the movement was of biological origin. It was Einstein's famous 1905 article on the subject that catapulted Brownian motion into twentieth century physics. Einstein showed that a molecular (as opposed to a continuum) model of water predicts the existence of the phenomenon that Brown observed. Interestingly, he predicted Brownian motion before learning about Brown's observations.[1]

Because it is virtually impossible to solve Newton's equations of motion for anything like the number of particles in a drop of water, Einstein adopted a statistical approach and showed that the evolution of the distribution of Brownian particles is governed by the heat equation. That is, the density of particles at each point follows the same physical law as the temperature at each point. Actually, from the physical point of view, this description of Einstein's paper throws out the baby with the wash. A physicist cannot talk about a one-size-fits-all *heat equation* any more than a one-size-fits-all *wave equation*; there are all-important constants which enter any physical equation. For the wave equation, the essential physical constant is the speed of light. In the case of the heat equation, there is the *diffusion constant*, and it was Einstein's formula for the diffusion constant which won his 1905 article its place in history. Namely, Einstein expressed the diffusion constant as the ratio of

[1] *On page 17 of* Dynamical theories of Brownian motion *(Princeton Univ. Press, 1967), Edward Nelson remarks, "It is sad to realize that despite all the hard work which had gone into the study of Brownian motion, Einstein was unaware of the existence of the phenomenon. He predicted it on theoretical grounds and formulated a correct quantitative theory of it." He quotes Einstein as saying, "My major aim…was to find facts which would guarantee as much as possible the existence of atoms of definite finite size."*

several physical quantities, one of which was Avogadro's number.[2]

It turns out that, with the exception of Avogadro's number, all these quantities, including the diffusion constant itself, were either known or measurable experimentally: Thus, his formula led to the first accurate determination of Avogadro's number.

If one ignores physics and analyzes Einstein's model from a purely mathematical standpoint, what Einstein was saying is summarized by the following three assertions about the way in which Brownian particles move:

1. *Brownian particles travel in such a way that the behavior over two different time intervals is independent. Thus, there is no way to predict future behavior from past behavior.*
2. *The particle is equally likely to move in any direction and the distance traversed by a Brownian particle during a time interval is on average proportional to the square root of the time.*
3. *The trajectories of Brownian particles are continuous.*

With reasonably standard results from the modern theory of probability, one can deduce from Einstein's three assumptions the conclusion that the distribution of Brownian particles evolves according to a heat equation. (The all-important diffusion constant is determined by the proportionality constant in (2).) Of course, in 1905, a mathematically satisfactory formulation of probability theory had yet to be given. Thus, Einstein's derivation was, mathematically speaking, rather primitive. Moreover, implicit in his model was an important mathematical challenge: *the verification that one can construct a distribution on the space of trajectories so that (1), (2), and (3) are satisfied.*[3]

*At Cambridge he discovered a different breed of student who accepted his eccentricities and thrived on intellectual discussion.*

At the turn of the century, the French school of analysis was hard at work creating the subject which we now call *measure theory* (i.e., the theory by which we assign volume to sets).[4]

The French school, especially E. Borel and H. Lebesgue, freed measure theory from its classical origins and made it possible to consider the problem of assigning *probabilities* to subsets of trajectories. However, in spite of their many magnificent achievements, neither Borel, Lebesgue, nor their disciples such as P. Lévy, S. Banach, M. Fréchet, and A. N. Kolmogorov, had been able to mathematically rationalize Einstein's model of Brownian motion. All of them were well aware of the essential problem, but none of them had been able to carry out the required construction. This was the problem that Wiener solved.

In hindsight, Wiener's strategy looks a little naive. In particular, he completely circumvented the issues on which more experienced mathematicians had foundered. In a marvelous demonstration of the power of optimism, he supposed that the desired assignment of probabilities could be made and asked how this assignment would look in a cleverly chosen coordinate system. He then turned the problem around and showed that the coordinate description leads to the existence of the desired assignment. (This general line of reasoning is fa-

*[2] Avogadro's number is a universal constant measuring the number of molecules in a gas per unit volume at a fixed pressure. It can also be defined as the number of atoms in one gram of hydrogen.*

*[3] Actually, Einstein's 1905 article was not the first one in which this problem appeared. Five years earlier, H. Poincaré's brilliant student L. Bachelier came to the conclusion that the fluctuation of prices on the Paris Bourse follows trajectories whose distribution satisfies (1), (2), and (3). It was not until the 1970s that the economics literature on this subject converged with the engineering and mathematical literature. The result is a much more sophisticated way to calculate risk in large financial markets, which has become an indispensible tool for loan, investment, and trading companies. Finally, one should remark that Bachelier, as distinguished from Einstein, really addressed the problem of computing the probability of nontrivial events which can be formulated only in the path-space context. The first physicist to address such problems was M. Smoluchowski, who used an approximation scheme based on random walks.*

*[4] Prior to their efforts, the only available theory was basically the one introduced by Archimedes, rediscovered by Fermat and Newton, and now forced on every calculus student. Of course, that theory had been tightened up by Cauchy, Riemann, and others, but it was still seriously deficient. For example, one could not show that the whole is the sum of its parts unless there were at most finitely many parts. In addition, although Riemann's theory served quite well in finite-dimensional contexts, there was no theory at all for infinite-dimensional spaces, like the space of all Brownian trajectories.*

miliar to anyone who has ever solved a problem by saying "let $x$ be the solution" and then found $x$ as a consequence of the properties which it must have.) Wiener's Gordian-knot solution to the problem enhances its appeal; and the assignment of probabilities at which Wiener arrived in "Differential Space" has, ever since, borne his name. It is called *Wiener measure*.

The importance of Wiener measure is hard to exaggerate; it represents what we now dutifully call a *paradigm*. For one thing, its very existence opened a floodgate and led Lévy, Kolmogorov, and others to create the theory of *stochastic processes*, thereby ushering in the modern theory of probability. In addition, Wiener measure is, in a sense which can be made very precise, as universal as the standard Gaussian (or normal) distribution on the real line: It is the distribution which arises whenever one carries out a central limit scaling procedure on path-space valued random variables.[5]

This is the underlying reason why Wiener measure arises as soon as one is studying a phenomenon which displays the properties in (1), (2), and (3) above. It is also the reason why, again and again, Wiener measure comes up in models of situations in which one is observing the net effect of a huge number of tiny contributions from mutually independent sources—as in the motion of a pollen particle, the Dow Jones average, or, as Wiener himself observed, the distortions in a signal transmitted over a noisy line.

Although his construction of Brownian motion was Wiener's premiere achievement during the period, it was not his only one. In a sequence of articles from 1923 through 1925, Wiener also looked at a fundamental problem in the theory of electrostatics. The problem was to decide what shape electrical conductor can carry a fixed charge. Zaremba had shown that certain conductors in the shape of spikes are unable to carry charge; they discharge spontaneously at the tip. (The reverse of this phenomenon is what makes a lightning rod work.) On the other hand, Zaremba had shown that cone-shaped conductors do hold their charge. In the mathematical model spontaneous discharge corresponds to an abrupt change, a *discontinuity*, in the voltage across the interface between the conductor and the surrounding medium. The electrical field has a constant voltage on the conductor, and the equilibrium is stable (no sparks) if the voltage is *continuous* across the interface.

Wiener described all shapes for which instability occurs and established a new framework for the entire subject of potential theory. In sharp contrast with many models in mathematical physics, he showed that the voltage in equilibrium is well defined mathematically, regardless of whether the conductor is stable or not. He then formulated a wholly original test, now known as the *Wiener criterion*, which determines at which points the voltage is discontinuous. A key step in Wiener's approach was to

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extend to arbitrary shapes a classical notion known as electrostatic capacity.[6]

He used a procedure that is analogous to, but more intricate than, the one invented by Lebesgue when he assigned a volume to regions for which there was no classical notion of volume. Indeed, Wiener's capacity is closely related to, but more subtle than, the measures used for fractals.[7]

Another topic on which Wiener worked during this period was what we now call *distribution theory* or the theory of *generalized functions*.

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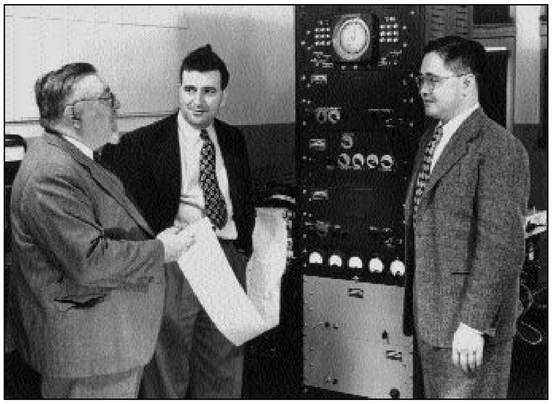
[5] *A full understanding of this universality came only in the 1950s and was provided by P. Lévy, R. H. Cameron, M. Donsker, P. Erdös, M. Kac, W. T. Martin, and I. E. Segal.*

[6] *The electrostatic capacity of a conductor can be defined as the total charge carried by the conductor in equilibrium when the voltage difference between the conductor and its surroundings is fixed at, say, one hundred volts.*

[7] *There is an amusing irony associated to Wiener's investigations into potential theory. As S. Kakutani discovered in the early 1940s, potential theory is related to Brownian motion in deep and wonderful ways. Wiener completely missed this beautiful and useful connection with his previous work. He might have been led in the*



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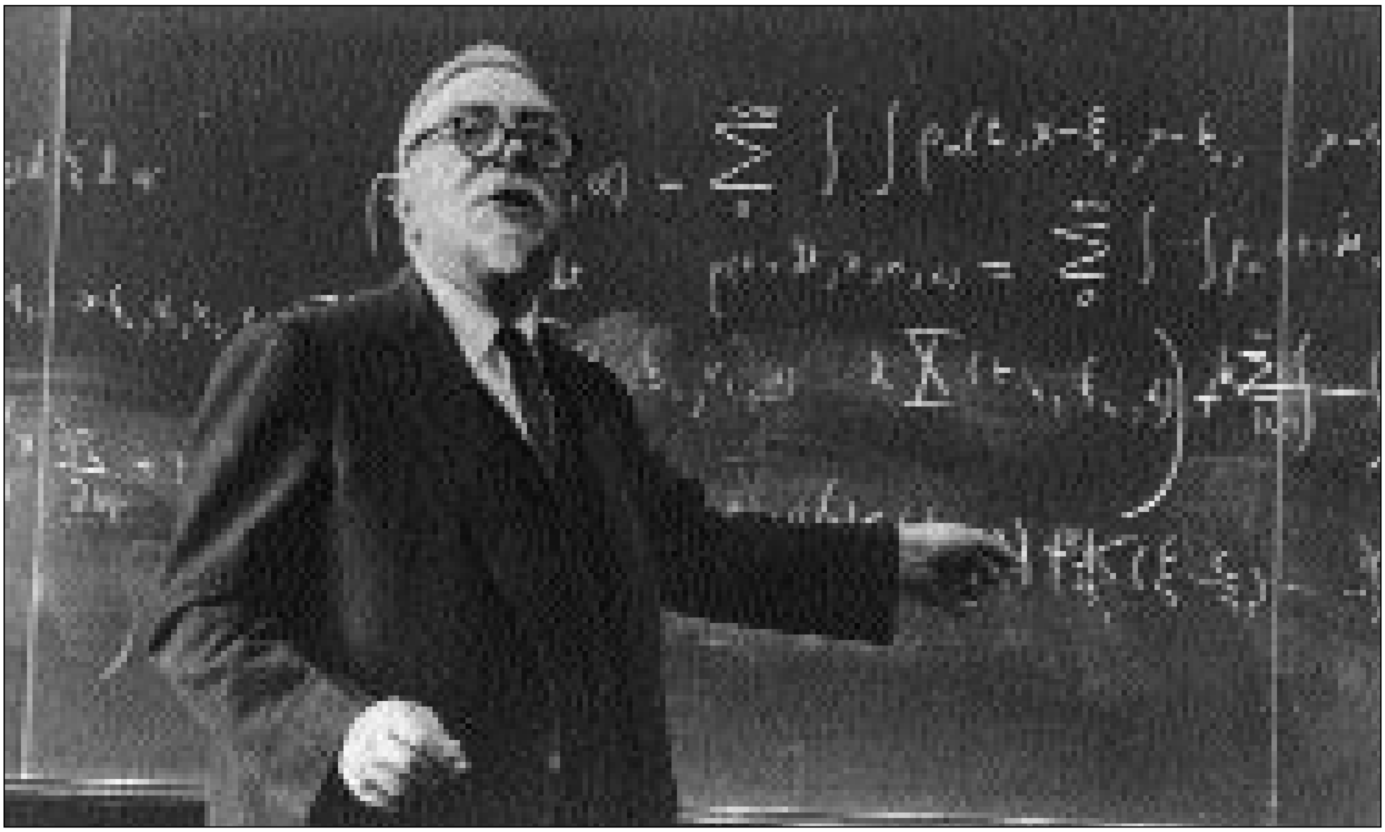
***Opposite left:*** **Norbert Wiener at age 7 in 1901.** ***Left:*** **Wiener, Jerome Wiesner, and Yuk W. Lee with the “MIT Autocorrelator” in 1949.** ***Below:*** **At MIT where he began his teaching career in 1919 and continued for the next 41 years.**



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Not long after Wiener arrived at MIT, Professor Jackson and other members of the electrical engineering department at MIT asked Wiener to develop a proper foundation for the Heaviside calculus, a calculus for solution of differential equations using Fourier and Laplace transforms. Heaviside’s calculus transforms a differential equation into an equation involving multiplication, as in $Ax = B$. To solve for $x$, we simply divide: $x = B/A$. The difficulty is that this easy formula for the solution then has to be transformed back into a meaningful statement about the solution to the original differential equation. This involves making sense out of the inverse of the Fourier-Laplace transform. Wiener undertook the description of how multiplication and division correspond to the operations of differentiation and integration. Laurent Schwartz, the father of the theory of distributions, acknowledges that Wiener’s treatment in 1926 anticipated all others by many years.[8]

Just as the physics of Brownian motion had stimulated Wiener to profound new mathematics, so the practical problem of processing elec-

*right direction if he had remembered that the distribution of Brownian paths is governed by the heat equation, and that temperature in steady state satisfies the Laplace equation, which is the very same equation satisfied by voltage in equilibrium.*

[8] *See p. 427 of L. Schwartz’s* Collected works II, *and p. 101 of* Norbert Wiener *by P. R. Masani, Birkhauser, 1990.*

trical signals led him to a deep extension of classical Fourier analysis. Fourier analysis consists of decomposing a periodic signal into a sum of pure sine waves. The fundamental formula of Fourier analysis, the Parseval formula, says that the total energy of the signal in each period is the sum of the energies of its pure waves. The collection of frequencies of the pure waves is known as the *spectrum* of the signal, and these come from a discrete list of values—the harmonics of a vibrating string. There is a similar fundamental formula due to Plancherel for the decomposition of nonperiodic waves that measures the total energy over all time. The spectrum of the signal is spread over the continuum of frequencies, and the formula measures the amount of energy of the signal concentrated in a given band of frequencies. The problem is that the signals that occur in practice in electrical systems do not fit into the frame of either of these theories. The signals are not periodic, and the spectrum is not confined to a special list, so that Fourier series are inadequate. On the other hand, the total energy over an infinite time period is infinite, so that Plancherel's theory does not apply. Wiener overcame this difficulty with what he named *generalized harmonic analysis*.[9]

> *Wiener believed that mathematicians cannot ignore the outside world and must both apply mathematics and bear the moral responsibility for applications.*

Wiener took as his starting place certain autocorrelation numbers, which compare the signal to the same signal with a time delay. These were precisely what could be measured in practice. Then, instead of dealing with total energy, Wiener considered the *average* energy of the signal over a long time interval. His theory was flexible enough to encompass both periodic signals and signals composed of a continuum of frequencies, such as "white noise".

One of the key ingredients in Wiener's generalized harmonic analysis was a new method to calculate limits of averages. His first step was to rephrase the problem so that it became one of determining when two different weighted averages are very close to each other. The recast problem fit into the general framework of so-called Tauberian theory, a theory to which Hardy and Littlewood had made several contributions. But instead of using some refinement of the techniques of his teachers, Wiener introduced a new approach that not only solved his own problem, but revealed the fundamental mechanism of all previous problems of this type.[10]

In his monograph on the subject[11] Wiener illustrates his ideas with an elegant proof of the Prime Number Theorem, one of the most beautiful applications of analysis to number theory.[12]

With the publication of his work on generalized harmonic analysis and Tauberian theorems, Wiener's reputation was at last established. In 1932 he was promoted to full professor at MIT and the following year he was elected to the National Academy of Sciences. In the same year he was awarded the Bôcher prize, a prize given every five years for the best work in analysis in the United States.

The major works outlined above by no means exhaust Wiener's intellectual activity. Throughout the 1930s he continued to expand on harmonic analysis, with the same engineering applications clearly in view. He wrote an influential book[13] with R.E.A.C. Paley and a seminal paper on integral equations with E. Hopf. He made excursions into quantum mechanics with Max Born and sorties into five-dimensional relativity (Kaluza-Klein theory) with Dirk Struik. In the late 1930s Wiener made a significant contribution to the mathematical foundations of statistical mechanics by extending G. D. Birkhoff's 1931 ergodic theorem. His 1938 paper, "The Homogeneous Chaos", which attempts to fathom nonlinear random phenomena, has de-

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[9] Generalized harmonic analysis, *Acta Math.* **55** *(1930), 117–258.*

[10] *Wiener's work led to I. M. Gelfand's far-reaching formulation of a notion of spectrum that can be used to analyze multiplication and division in any algebraic system.*

[11] Tauberian theorems, *Ann. of Math.* **33** *(1932), 1–100.*

[12] *The Prime Number Theorem says that the probability that a number N is prime is* $1/\ln N$, *where* ln *is the natural logarithm. The reason why the Prime Number Theorem is related to harmonic analysis is that the Riemann zeta function is the Mellin transform of (a variant of) the counting function for the number of primes. The Mellin transform is just the Fourier transform disguised by a logarithmic change of coordinates.*

[13] Fourier transforms in the complex domain, *Amer. Math. Soc. Colloq. Publ., vol. 19, Amer. Math. Soc., Providence, RI, 1934.*

scendents in constructive quantum field theory, under the name "Wick ordering".

### World War II, Feedback, and Filters

In 1933 Wiener became acquainted with Arturo Rosenblueth, a Mexican physiologist who was leading a series of interdisciplinary seminars at the Harvard Medical School. They hit it off well and began a long association during which Wiener's ideas on the relationship between mechanical and physiological systems—particularly in regard to the role of feedback—came to fruition. It appears that his interaction with Rosenblueth also set in motion the train of thought which would evolve into *cybernetics*. Thus, from an intellectual and scientific standpoint, their collaboration was an enormous success. In addition, judging from the warmth with which Wiener writes of him, Rosenblueth became the closest friend of his adult life.

The concept of a *feedback loop* was already familiar to James Watt in the eighteenth century, and today it so deeply embedded in our thought processes that we hardly recognize it. An everyday example of a feedback loop is the one connecting a furnace to a thermostat. The furnace puts out heat, raising the temperature of the room. The thermostat senses the temperature, and if it gets too low, the thermostat completes a circuit and ignites the furnace. The furnace then continues to pump out heat until the temperature gets too high, at which point the thermostat breaks the circuit, and the furnace shuts down. In this way, the output of the furnace is fed back into the input.[14]

What fascinated Wiener were unstable feedback mechanisms. Most of us know the difficulty of carrying a too-full bowl of soup to the dinner table: The soup begins to slosh and any attempt to settle it by tilting (negative feedback) only makes matters worse. Wiener and Rosenblueth proposed to model certain muscle spasms (intention tremors) using an unstable feedback loop. Later they used the same principles to study the heart muscle.

With the outbreak of World War II, Wiener had to defer these investigations. Confronted by what appeared to be the imminent collapse of European civilization, Wiener, like many scientists, searched for a way to contribute to the war effort. The problem he eventually chose was that of aiming antiaircraft guns. This was a much more sophisticated problem than the ones he had worked on in World War I. Airplanes had become much faster and more dangerous, and so the human gunner had to be assisted by a machine. Moreover, it was no longer sensible to aim directly at the plane: by the time the shell got there, the plane would have moved on. The problem was therefore one of *prediction*. That is, one had to determine the plane's position by radar signals and *predict* its future trajectory. Since it was clear that there was no hope of making a perfect prediction, Wiener decided to adopt a statistical approach. In other words, he devised a statistical model in which he could formulate precisely what it means to *maximize the probability of success*.

*The main threads of his mathematical work were fundamental to probability theory and harmonic analysis, and these threads are woven into the fabric of contemporary mathematics.*

A central difficulty addressed by Wiener's statistical model was that if one tries to control the action of the gun too closely from the radar data, measurement errors can cause the gun to go into wild oscillations. Human gunners have no trouble adjusting to imperfect measurements, but a machine had to be designed specifically to prevent instabilities. Wiener compensated for the imperfection of the radar data by averaging them to remove *noise* (random measurement errors). When the data are averaged over time, the oscillations are dampened. His ideas were closely related to those he had about stabilizing unstable feedback loops. Of course, one has to be careful lest the averaging obliterate useful information. The whole point was to make a judicious choice of averaging procedure that retained as much information as possible.

In 1942 Wiener's collaborator, Julian Bigelow,[15] built a prototype to track an airplane

[14] *This sort of feedback is called negative feedback because the thermostat reverses the action of the furnace.*

for ten seconds and predict its location twenty seconds later. Sad to say, Wiener and Bigelow's efforts did not hasten the end of the war. It was only after the war that improvements in the speed and accuracy of airplane and radar equipment made systematic filtering and prediction devices very important. On the other hand, Wiener's ideas had ramifications far beyond their original motivation. On being confronted with a stream of data embedded in noise, the antiaircraft predictor is faced with the same problem as the communication engineer, who must send or extract a message from a noisy channel. In both cases it is possible to design a *filter* to exclude the noise, which is the engineer's term for what Wiener did. Filtering is any strategy to filter out the effects of random vibration or static from a mechanical or electrical system. Filters are needed in all sorts of devices, from stereo equipment to aircraft instrumentation.

Under the assumptions he made, Wiener's solution to the prediction and filtering problems was the best possible in a sense that is mathematically precise. Independently, at essentially the same time, A. N. Kolmogorov, the great Russian probabilist, came up with a similar mathematical theory. Thus, Kolmogorov and Wiener developed the first systematic approach to the design of filters. However, their assumptions are not realistic in many applications. In technical jargon, their strategy is designed for random disturbances which are linear functions of white noise; it does not do a good job when the disturbances are nonlinear functions of white noise. Later on, Wiener addressed nonlinear problems with what he called the theory of *homogeneous chaos*, but neither Wiener nor Kolmogorov nor anyone else has achieved the kind of comprehensive success with nonlinear filtering that he did in the linear case.

Wiener wrote up these results in a 1942 report entitled "The interpolation, extrapolation of linear time series and communication engineering" The book was dubbed "the yellow peril" because of its yellow covers and its frightening mathematics. Wiener spent over a year working intensively on this report, only to have it be classified. Given Wiener's irrepressible urge to talk about his work and his desire to pursue it further, the classification was intolerable. From then on he frequently railed against military secrecy and proclaimed its incompatibility with free scientific inquiry.

Wiener retired from MIT in 1960 and died in 1964. The main threads of his mathematical work were fundamental to probability theory and harmonic analysis, and these threads are woven into the fabric of contemporary mathematics. His creative energy will continue to have a profound impact on mathematics, science, and technology for the century to come.

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*15 Bigelow was subsequently hired by John von Neumann to build the first programmable computer, the ENIAC.*